Aspects of determining f_{B_s}: scaling and power-law divergences

Presented by S. Collins, UKQCD Collaboration[a] *

[a]Dept. of Physics an d Astronomy, Glasgow University, Glasgow, G12 8QQ, Scotland

We present *preliminary* results for the decay constant of the B_s meson, f_{B_s}, at three values of $\beta = 5.7$, 6.0 and 6.2 using NRQCD and clover fermions for the heavy and light quarks respectively. As a consistency check the decay constant has also been extracted from the axial-vector current at finite momentum. In addition, we discuss the cancellation of $O(\alpha/(aM_0))$ terms and the remaining uncertainty in f_{B_s} from higher order divergences.

1. Simulation details

For the heavy quark we use an NRQCD action consistent to $O(1/M_0^2)$:

$$S = Q^\dagger(\Delta_t + H_0 + \delta H)Q \tag{1}$$

where,

$$H_0 = -\frac{\Delta^{(2)}}{2M_0} \tag{2}$$

$$\begin{aligned}\delta H = & -c_1\frac{\sigma\cdot\mathbf{B}}{2M_0} + c_2\frac{(\mathbf{\Delta}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{\Delta})}{8M_0^2} \\ & -c_3\frac{\sigma\cdot(\mathbf{\Delta}\times\mathbf{E} - \mathbf{E}\times\mathbf{\Delta})}{(8M_0^2)} - c_4\frac{(\mathbf{\Delta}^{(2)})^2}{8M_0^3} \\ & +c_5\frac{a^2\mathbf{\Delta}^{(4)}}{24M_0} - c_6\frac{a(\mathbf{\Delta}^{(2)})^2}{16nM_0^2}.\end{aligned} \tag{3}$$

The $O(1/M_0^3)$ correction to the kinetic energy (expected to be the largest contribution from this order) and the first two discretisation corrections are also included. We implement tadpole improvement throughout, using the plaquette definition of u_0, and set $c_i = 1$.

For the light quark we use the clover action with the tadpole improved value of c_{SW} at $\beta = 5.7$ and 6.0 and the non-perturbatively determined value at $\beta = 6.2$; $c_{SW} = 1.61$ as determined by the Alpha collaboration [1], compared to $c_{SW} = 1.48$ using tadpole-improvement. The configurations at $\beta = 5.7$ and the configurations and light quark propagators at $\beta = 6.2$ were generously provided by the UKQCD collaboration.

*In collaboration with C. T. H. Davies and J. Hein, Glasgow Univ.; J. Shigemitsu, Ohio-state Univ.; C. Morningstar, San Diego; A. Ali Khan, Tsukuba.

β	V	N	$a^{-1}(m_\rho)$ (GeV)	aM_0^b
5.7	$12^3\times 24$	278	1.14(3)	~ 4.2
6.0	$16^3\times 48$	102	1.92(7)	~ 2.2
6.2	$24^3\times 48$	144	2.63(9)	~ 1.6

Table 1
The simulation details. The errors on a^{-1} include statistical errors and those due to the chiral extrapolation of m_ρ. N denotes the number of configurations.

The light quark mass is fixed to the strange quark mass using the K meson mass, with the uncertainty in this determination estimated by fixing m_q using the ϕ. Further simulation details for the 3 ensembles are given in table 1.

The pseudoscalar decay constant is defined as

$$\langle 0|\, A_\mu\, |PS\rangle_{QCD} = p_\mu f_{PS} \tag{4}$$

in Euclidean space. On the lattice matching factors C_i relate the lattice operators to the current in full QCD. For the zeroth component of the current to $O(\alpha/M)$:

$$\langle A_0\rangle_{QCD} = \sum_j C_j(\alpha, aM_0)\langle J_L^i\rangle \tag{5}$$

where,

$$O(1) : J_L^{(0)} = \bar{q}\gamma_5\gamma_0 Q \tag{6}$$

$$O(\frac{1}{M}) : J_L^{(1)} = -\frac{1}{2M_0}\bar{q}\gamma_5\gamma_0(\gamma\cdot\mathbf{D})Q \tag{7}$$

$$O(\frac{\alpha}{M}) : J_L^{(2)} = \frac{1}{2M_0}\bar{q}(\gamma\cdot\overleftarrow{\mathbf{D}})\gamma_5\gamma_0 Q. \tag{8}$$

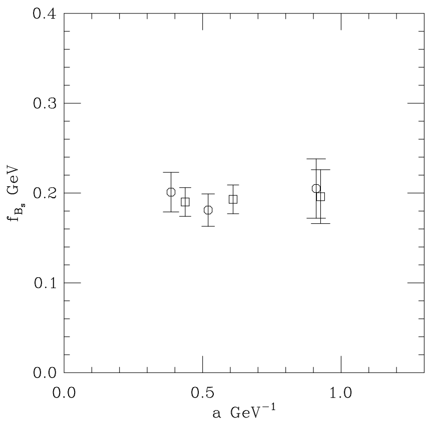

Figure 1. Preliminary results for f_{B_s} as a function of the lattice spacing. Our results (circles) are compared to those from JLQCD (squares) [6]. All errors include statistical and systematic uncertainties added in quadrature.

The $O(a\alpha)$ discretisation error in the current is removed by defining [2],

$$J_L^{0,imp} = J_L^{(0)} + C_A J_L^{(disc)} \quad (9)$$

$$J_L^{(disc)} = a\bar{q}(\gamma\cdot \overleftarrow{\mathbf{D}})\gamma_5\gamma_0 Q \quad (10)$$

The C_is have been calculated to 1-loop [2], however, the q^* (which also depends on aM_0) at which the strong coupling, α, is computed is not yet known. Thus, we average the results obtained using $aq^* = 1.0$ and π. Note that in the static limit $aq^* \sim 2$ [3] for Wilson light fermions.

The results at $\beta = 5.7$ and 6.0 have appeared previously in [4] and [5] respectively. Further details of our methods and analysis can be found in these references.

2. Scaling of f_{B_s}

Our values for f_{B_s}, calculated to $O(\alpha/M)$, are presented in figure 1 as a function of a^{-1}. The results are consistent with scaling, and $f_{B_s} \sim$ 200 MeV. Nice agreement is found with the results of the JLQCD collaboration [6], also shown in the figure. This group uses the string tension to set a, and an $O(1/M_0)$ NRQCD action. However, this is unlikely to affect the comparison significantly.

Table 2 details how the errors are estimated.

Source	$\beta = 5.7$	6.0	6.2
statistical	3	3	2
disc. $O((a\Lambda_{QCD})^2)$	**13**	4	2
pert. $O(\alpha^2, \alpha/(aM)^2)$	**13**	**8**	**9**
NRQCD $O(1/M^2)$	1	1	1
κ_s	+4	+4	+4
$a^{-1}(m_\rho)$	3	4	3
Total	**19**	**11**	**11**

Table 2
Estimates of the statistical and main systematic errors, in percent, in our values for f_{B_s}

Note that $a^{-1} \sim 1-2.6$ GeV is the range in which the NRQCD action and clover light fermions can be applied to the B meson. For coarser values of a, the discretisation errors from the light quark action (and gauge action) increase rapidly, as do the $O(\alpha^2)$ perturbative errors. Conversely, if a becomes too fine, the discretisation errors are under control but the $O(\alpha/(aM)^2)$ perturbative errors increase dramatically as aM_0^b drops below 1. Overall, the perturbative errors are the main source of uncertainty, although at $\beta = 5.7$ the discretisation errors are of equal magnitude.

3. f_{B_s} extracted at finite momentum

In order to investigate momentum dependent discretisation errors in the decay constant we computed the ratio

$$\langle J_L^{(0)}\rangle_{\vec{p}} / \langle J_L^{(0)}\rangle_{\vec{0}} = \sqrt{E(p)/M} \quad (11)$$

where $\langle J_L^{(i)}\rangle_{\vec{p}} = f^i\sqrt{E(\vec{p})}$ (without renormalisation). The RHS of equation 11 is a slowly varying function of $|\vec{p}|$, which is close to 1 for the range of momenta we studied (up to 1.5 GeV at $\beta = 6.2$). Our results, presented in table 3, are in agreement with this expectation at $\beta = 6.0$ and 6.2, i.e. the momentum dependent discretisation errors in f_{B_s} are not significant. However, at $\beta = 5.7$ a $10-20\%$ deviation from 1, is seen as $|\vec{p}|$ increases, although this is within the magnitude expected for $O((ap)^2)$ discretisation errors.

4. Power-law divergences

Matrix elements in NRQCD beyond zeroth order diverge as $aM_0 \rightarrow 0$. In the case of f_B

$(f^0\sqrt{E(\vec{p})})/(f^0\sqrt{M})$				
β	$n^2=1$	2	3	4
5.7	0.94(1)	0.88(1)	0.84(1)	0.82(2)
6.0	1.01(1)	1.01(3)	-	-
6.2	1.00(2)	1.02(3)	1.03(4)	1.07(6)

Table 3
The decay constant extracted at finite momentum for aM_0 close to aM_0^b. $|\vec{p}| = 2n\pi/(aL)$, where $n = 0, 1, \sqrt{2}\ldots$ and L is the spatial extent of the lattice.

there are unphysical, ultra-violet, contributions to $\langle J_L^{(i)}\rangle, i > 0$, which are cancelled order by order in perturbation theory by terms appearing in the perturbative coefficients. In general, simulations are performed using aM_0>1 and so the unphysical contributions are not expect to be large and their cancellation should be under control.

Considering equation 5 in more detail (see [2] for definitions),

$$\begin{aligned}\langle A_0\rangle_{QCD} \quad &= \quad (1+\alpha\rho_0)\langle J_L^{(0)}\rangle + \langle J_L^{(1)}\rangle \\ &+\alpha\rho_1\langle J_L^{(1)}\rangle + \alpha\rho_2\langle J_L^{(2)}\rangle \\ &+\alpha\rho_{disc}\langle J_L^{(disc)}\rangle \end{aligned} \tag{12}$$

The explicit contributions to ρ_0 are

$$\rho_0 = [B_0 - \frac{1}{2}(C_q + C_Q) - \zeta_{00} - \zeta_{10}]. \tag{13}$$

The lowest order divergent contribution to the current is $O(\alpha/(aM))$, which appears through the tree-level term $\langle J_L^{(1)}\rangle$ and is **cancelled** by the mixing term $\alpha\zeta_{10}\langle J_L^{(0)}\rangle$, where ζ_{10} is the renormalisation due to the mixing between $J_L^{(0)}$ and $J_L^{(1)}$. The remaining divergent contributions, $O(\alpha/(aM)^2, \alpha^2/(aM))$ etc, appearing in equation 12, are cancelled at higher orders; the uncertainty in f_{B_s} due to these remaining terms is well within our estimates of the systematic uncertainties in table 2.

In fact a large part of $\langle J_L^{(1)}\rangle$ is unphysical, $\alpha\zeta_{10}\langle J_L^{(0)}\rangle/\langle J_L^{(1)}\rangle \sim 0.5 - 0.6$ for all βs and aM_0 for $aq^* = 2.0$. However, once the $O(\alpha/(aM))$ contribution to $\langle J_L^{(1)}\rangle$ is cancelled, the scaling behaviour of this term improves, as shown in figure 2, suggesting the remainder is physical.

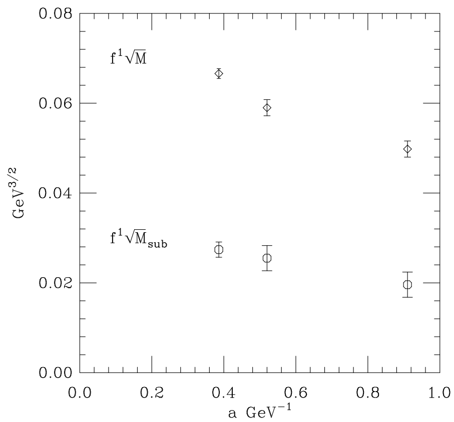

Figure 2. The contribution to the decay constant from the tree-level current correction before and after the subraction of the $\alpha\zeta_{10}\langle J_L^{(0)}\rangle$, $aq^* = 2.0$.

However, $\langle J_L^{(1)}\rangle - \alpha\zeta_{10}\langle J_L^{(0)}\rangle \sim 0.025\text{GeV}^{3/2}$ is small, (cf $f^0\sqrt{M} \sim 0.500\text{GeV}^{3/2}$), and of the order of the higher order perturbative and NRQCD corrections. Hence, we cannot determine the size of the physical part of $\langle J_L^{(1)}\rangle$ reliably. We emphasise this does not lead to a significant uncertainty in f_{B_s}, nor in the slope of the decay constant with $1/M$, which are dominated by $\langle J_L^{(0)}\rangle$ and the corresponding perturbative coefficient (without the ζ_{10} term).

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